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A new geometrical nonlinear laminated theory for large deformation analysis

H.-F. Tan*, Z.-H. Tian, X.-W. Du

Research Laboratory of Composite Materials, Harbin Institute of Technology, Harbin, 150001, People's Republic of China

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Abstract

A six-variable geometrical nonlinear shear deformation laminated theory is presented in which normal stress and strain distribution can be calculated. By considering some affective factors that were neglected under the finite deformation condition, an improved Von Karman geometrical nonlinear deformation-strain relation is used for large deformation analysis. By analyzing the bending problem of laminated plates, and by comparing it with 3-D elasticity solutions and J.N. Reddy five-variable simple higher-order shear deformation laminated theory, we can come to a conclusion that a satisfying precision of the calculation studied in this paper has been achieved, which shows that it is especially suitable for application of the calculation in the condition of a large deformation and the laminated thick plate analysis. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

With the increasing use of composite materials in thick laminated form and large deformation analysis, the need for advanced methods of analysis is obvious. For such laminated systems, the components of stress and strain transverse in the plane of laminate strongly influence the behavior. Many different higher-order laminated plate theories have been proposed which are intended to improve upon the classical laminated plated theory by accounting for the effects of transverse components of strain in the plates (Lo, 1977; Reddy, 1984; Chia, 1988; Noor, 1989; Shu, 1994). A higher-order laminated theory for flexural behavior of laminated plates was proposed by Lo (1977), where eleven equilibrium equations were obtained for the determination of the eleven generalized displacement coefficients in the assumed displacement fields. A simple higher-order shear deformation theory of laminated plates was developed by Reddy (1984), which contained the same dependent unknowns as in the first-order shear deformation theory of Whitney and Pagano, but account for the parabolic

* Corresponding author. Tel.: +86-451-641-6505; fax: +86-451-622-1048.
E-mail address: duxw@hope.hit.edu.cn (H.-F. Tan)

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distribution of the transverse shear strains through the thickness of the plate. Meanwhile Reddy assumed that w is not a function of the thickness coordinate. However the use of three-dimensional models for predicting the response characteristics of laminated anisotropic plates with complicated geometry is computationally expensive and, therefore, is not feasible for practical composite plates. On the other hand the two-dimensional theories are adequate for predicting the gross response characteristics of medium-thick laminated plates, but they are not adequate for the accurate prediction of normal stresses and deformations.

By considering some affective factors that were neglected under the finite deformation condition, an improved Von Karman deformation-strain relation is used for large deformation analysis. The discussions of this paper focus on developing a three-dimensional geometrical nonlinear laminated theory in which normal stress and strain distribution can be calculated. A relatively simple model for laminated plates is provided to proceed geometrical nonlinear analysis and avoid the complicated compute.

2. Displacement field assumption

We begin with the displacement field

$$u(x, y, z,) = u^0(x, y) + z\psi_x(x, y) + z^2\zeta_x(x, y) + z^3\phi_x(x, y)$$

$$v(x, y, z) = v^0(x, y) + z\psi_y(x, y) + z^2\zeta_y(x, y) + z^3\phi_y(x, y)$$

$$w(x, y, z) = w^0(x, y) + z^2\zeta_z(x, y) \tag{1}$$

Where u^0, v^0, and w^0 denote the displacements of a point (x, y) on the midplane, and ψ_x and ψ_y are the rotations of normal to midplane about y and x axes, respectively. The functions ζ_x, ζ_y, ϕ_x, ϕ_y, and ζ_z will be determined by using the condition that transverse shear stress, and $\sigma_{xz} = \sigma_5$, and $\sigma_{yz} = \sigma_4$ vanish on the plate top and bottom surfaces.

$$\sigma_4\left(x, y, \pm\frac{h}{2}\right) = 0, \quad \sigma_5\left(x, y, \pm\frac{h}{2}\right) = 0 \tag{2}$$

For orthotropic plates and plates laminated of orthotropic layers, these conditions are equivalent to the requirement that corresponding strains be zero on the surface, we have

$$\varepsilon_4 = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \psi_y + 2z\zeta_y + 3z^2\phi_y + \frac{\partial w^0}{\partial y} + z^2\frac{\partial \zeta_z}{\partial y}$$

$$\varepsilon_5 = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \psi_x + 2z\zeta_x + 3z^2\phi_x + \frac{\partial w^0}{\partial x} + z^2\frac{\partial \zeta_z}{\partial x} \tag{3}$$

By setting

$$\varepsilon_4\left(x, y, \pm\frac{h}{2}\right) = 0, \quad \varepsilon_5\left(x, y, \pm\frac{h}{2}\right) = 0$$

We obtain

$$\zeta_x = 0 \quad \phi_x = -\frac{4}{3h^2}\left(\frac{\partial w^0}{\partial x} + \psi_x\right) - \frac{1}{3}\frac{\partial \zeta_z}{\partial x}$$

and

$$\zeta_y = 0 \quad \phi_y = -\frac{4}{3h^2}\left(\frac{\partial w^0}{\partial y} + \psi_y\right) - \frac{1}{3}\frac{\partial \zeta_z}{\partial y} \tag{4}$$

The displacement field in eqn (1) becomes

$$u(x, y, z) = u^0(x, y) + z\left\{\psi_x(x, y) - \frac{z^2}{3}\left[\frac{4}{h^2}\left(\frac{\partial w^0}{\partial x} + \psi_x\right) + \frac{\partial \zeta_z}{\partial x}\right]\right\}$$

$$v(x, y, z) = v^0(x, y) + z\left\{\psi_y(x, y) - \frac{z^2}{3}\left[\frac{4}{h^2}\left(\frac{\partial w^0}{\partial y} + \psi_y\right) + \frac{\partial \zeta_z}{\partial y}\right]\right\}$$

$$w(x, y, z) = w^0(x, y) + z^2\zeta_z(x, y) \tag{5}$$

We use the same thought chosen by Reddy (1984) and Shu (1994) to simplify the complex displacement fields given by Lo (1977). Moreover we add the three-dimensional term to displacement field eqn (5) by supplementing ζ_z.

3. Geometrical nonlinear relations

The nonlinear theory of laminated anisotropic plates always based on the von Karman plate theory. The theory is known to be able to predict the global behavior accurately. However, it is not accurate enough to predict large deformation behavior of large deflections and large strains. For the purpose of three-dimensional and large deformation analyses here, give some modifications to the von Karman geometrical nonlinear plate theory.

Based on the definition of Green strain in large deformation:

$$\varepsilon_{ij} = \frac{1}{2}\left[\frac{\partial u_j}{\partial X_i} + \frac{\partial u_i}{\partial X_j} + \frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\right] \tag{6}$$

Some assumptions are presented here.

$$\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 \Longrightarrow \frac{1}{2}\left(\frac{\partial u^0}{\partial x}\right)^2, \quad \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \Longrightarrow \frac{1}{2}\left(\frac{\partial w^0}{\partial x}\right)^2, \quad \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 \Longrightarrow 0$$

$$\frac{1}{2}\left(\frac{\partial v}{\partial y}\right)^2 \Longrightarrow \frac{1}{2}\left(\frac{\partial v^0}{\partial y}\right)^2, \quad \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \Longrightarrow \frac{1}{2}\left(\frac{\partial w^0}{\partial y}\right)^2, \quad \frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^2 \Longrightarrow 0 \tag{7}$$

We get the strain-displacement relations,

$$\varepsilon_1 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial u^0}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w^0}{\partial x}\right)^2, \quad \varepsilon_2 = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial v^0}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial w^0}{\partial y}\right)^2 \tag{8}$$

Also consider that normal strain is linear distribution along the thickness.

$$\varepsilon_3 = \frac{\partial w}{\partial z} \tag{9}$$

The following terms in eqn (10) are neglected when accounting for shear strain-displacement relations.

$$\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}, \quad \frac{\partial v}{\partial x}\frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial z}\frac{\partial u}{\partial y}, \quad \frac{\partial v}{\partial z}\frac{\partial v}{\partial y}, \frac{\partial u}{\partial x}\frac{\partial u}{\partial z}, \quad \frac{\partial v}{\partial x}\frac{\partial v}{\partial z} \tag{10}$$

and also

$$\frac{\partial w}{\partial z}\frac{\partial w}{\partial y} \Longrightarrow 0, \quad \frac{\partial w}{\partial x}\frac{\partial w}{\partial z} \Longrightarrow 0, \quad \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \Longrightarrow \frac{\partial w^0}{\partial x}\frac{\partial w^0}{\partial y} \tag{11}$$

So we have the transverse shear strain-displacement relations

$$\varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w^0}{\partial x}\frac{\partial w^0}{\partial y}\right), \quad \varepsilon_{23} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right), \quad \varepsilon_{31} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \tag{12}$$

Expressing the strain-displacement relation with midplane displacements, we have,

$$\varepsilon_1 = \left[\frac{\partial u^0}{\partial x} + \frac{1}{2}\left(\frac{\partial u^0}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w^0}{\partial x}\right)^2\right] + z\frac{\partial \psi_x}{\partial x} - \frac{z^3}{3}\left[\left(\frac{4}{(h)^2}\frac{\partial^2 w^0}{\partial x^2} + \frac{\partial \psi_x}{\partial x}\right) + \frac{\partial \zeta_z}{\partial x}\right]$$

$$\varepsilon_2 = \left[\frac{\partial v^0}{\partial y} + \frac{1}{2}\left(\frac{\partial v^0}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial w^0}{\partial y}\right)^2\right] + z\frac{\partial \psi_y}{\partial y} - \frac{z^3}{3}\left[\left(\frac{4}{(h)^2}\frac{\partial^2 w^0}{\partial y^2} + \frac{\partial \psi_y}{\partial y}\right) + \frac{\partial \zeta_z}{\partial y}\right]$$

$$\varepsilon_3 = 2z\zeta_z$$

$$\varepsilon_4 = 2\varepsilon_{23} = \left(\psi_y + \frac{\partial w^0}{\partial y}\right) - \frac{4z^2}{(h)^2}\left(\frac{\partial w^0}{\partial y} + \psi_y\right)$$

$$\varepsilon_5 = 2\varepsilon_{31} = \left(\psi_x + \frac{\partial w^0}{\partial x}\right) - \frac{4z^2}{(h)^2}\left(\frac{\partial w^0}{\partial x} + \psi_x\right)$$

$$\varepsilon_6 = 2\varepsilon_{12} = \left(\frac{\partial v^0}{\partial x} + \frac{\partial u^0}{\partial y} + \frac{1}{2}\frac{\partial^2 w^0}{\partial x \partial y}\right) + z\left(\frac{\partial \psi_y}{\partial x} + \frac{\partial \psi_x}{\partial y}\right) + z^2\left[-\frac{4}{h^2}\left(2\frac{\partial^2 w^0}{\partial x \partial y} + \frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right) - \frac{2}{3}\frac{\partial^2 \zeta_z}{\partial x \partial y}\right] \tag{13}$$

Eqn (13) expresses the strains associated with the displacements in eqn (5). is different from the von Karman geometrical nonlinear plate theory. Some modifications are made here.

1. The square of midplane displacements cannot be neglected, i.e. $1/2\ (\partial u^0/\partial x)^2$ and $1/2\ (\partial v^0/\partial y)^2$ are retained.
2. Rotations $\partial u/\partial y$ and $\partial v/\partial x$ are also considered.
3. Normal strain-displacement relation is considered to be linear.

4. Constitutive equations

Upon transformation, the lamina constitutive equations can be expressed in terms of stresses and strains in the plate (laminated) coordinates as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} Q_{11}, & Q_{12}, & Q_{13}, & 0, & 0, & Q_{16} \\ Q_{12}, & Q_{22}, & Q_{23}, & 0, & 0, & Q_{26} \\ Q_{13}, & Q_{23}, & Q_{33}, & 0, & 0, & Q_{36} \\ 0, & 0, & 0, & Q_{44}, & Q_{45}, & 0 \\ 0, & 0, & 0, & Q_{44}, & Q_{55}, & 0 \\ Q_{16}, & Q_{26}, & Q_{36}, & 0, & 0, & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} \tag{14}$$

Where Q_{ij} are the transformed material constants.

5. Six-variable equilibrium equations

The principle of virtual displacements was used to derive the equilibrium equations appropriate for the displacement field in eqn (5) and constitutive equation in eqn (14). The principle of virtual displacements can be stated in analytical form as (Reddy, 1984).

$$\delta A = \int_{-h/2}^{h/2} \int_R (\sigma_1 \delta\varepsilon_1 + \sigma_2 \delta\varepsilon_2 + \sigma_3 \delta\varepsilon_3 + \sigma_4 \delta\varepsilon_4 + \sigma_5 \delta\varepsilon_5 + \sigma_6 \delta\varepsilon_6)\, \mathrm{d}A\, \mathrm{d}z$$

$$= \int_R \Bigg\{ N_1 \frac{\partial \delta u^0}{\partial x} + M_1 \frac{\partial \delta\psi_x}{\partial x} + P_1 \left[-\frac{4}{3h^2}\left(\frac{\partial^2 \delta w^0}{\partial x^2} + \frac{\partial \delta\psi_x}{\partial x} \right) - \frac{1}{3}\frac{\partial^2 \delta\zeta_z}{\partial x^2} \right]$$

$$+ N_1 \frac{\partial u^6}{\partial x}\frac{\partial \delta u^6}{\partial x} + N_1 \frac{\partial w^0}{\partial x}\frac{\partial \delta w^0}{\partial x}$$

$$+ N_2 \frac{\partial \delta v^0}{\partial y} + M_2 \frac{\partial \delta\psi_y}{\partial y} + P_2 \left[-\frac{4}{3h^2}\left(\frac{\partial^2 \delta w^0}{\partial y^2} + \frac{\partial \delta\psi_y}{\partial y} \right) - \frac{1}{3}\frac{\partial^2 \delta\zeta_z}{\partial y^2} \right]$$

$$+ N_2 \frac{\partial v^0}{\partial y}\frac{\partial \delta v^0}{\partial y} + N_2 \frac{\partial w^0}{\partial y}\frac{\partial \delta w_0}{\partial y} + N_6\left(\frac{\partial \delta u^0}{\partial y} + \frac{\partial \delta v^0}{\partial x} \right) + M_6\left(\frac{\partial \delta\psi_x}{\partial y} + \frac{\partial \delta\psi_y}{\partial x} \right)$$

$$+N_6\frac{\partial\delta w^0}{\partial x}\frac{\partial w^0}{\partial y}+N_6\frac{\partial w^0}{\partial x}\frac{\partial\delta w^0}{\partial y}$$

$$+P_6\left[-\frac{4}{3h^2}\left(2\frac{\partial^2\delta w^0}{\partial x\partial y}+\frac{\partial\delta\psi_x}{\partial y}+\frac{\partial\delta\psi_y}{\partial x}\right)-\frac{2}{3}\frac{\partial^2\delta\zeta_z}{\partial x\partial y}\right]$$

$$+Q_2\left(\partial\psi_y+\frac{\partial\delta w^0}{\partial y}\right)+R_2\left[-\frac{4}{h^2}\left(\frac{\partial\delta w^0}{\partial y}+\delta\psi_y\right)\right]$$

$$+Q_1\left(\delta\psi_x+\frac{\partial\delta w^0}{\partial x}\right)+R_1\left[-\frac{4}{h^2}\left(\frac{\partial\delta w^0}{\partial x}+\delta\psi_x\right)\right]+2M_3\delta\zeta_z\Bigg\}\,\mathrm{d}x\,\mathrm{d}y \tag{15}$$

We use the strains from eqn (13) and the following definitions of stress resultants.

$$(N_i, M_i, P_i)=\int_{-h/2}^{h/2}\sigma_i\left(1, z, z^3\right)\mathrm{d}z \qquad (Q_1, R_1)=\int_{-h/2}^{h/2}\sigma_5\left(1, z^2\right)\mathrm{d}z \quad (i=1, 2, 6)$$

$$(Q_2, R_2)=\int_{-h/2}^{h/2}\sigma_4\left(1, z^2\right)\mathrm{d}z \qquad M_3=\int_{-h/2}^{h/2}z\sigma_z\,\mathrm{d}z \tag{16}$$

Collecting the coefficients of

$$\delta u^0, \delta v^0, \delta w^0, \delta\psi_x, \delta\psi_y, \delta\zeta_z, \delta\frac{\partial w^0}{\partial x}, \delta\frac{\partial\zeta_x}{\partial x},$$

we obtain the following equilibrium equations in the domain:

$$\delta u^0\colon N_{1,x}+N_{6,y}+\underline{\frac{\partial}{\partial x}\left(N_1\frac{\partial v^0}{\partial x}\right)}=0$$

$$\delta v^0\colon N_{6,x}+N_{2,y}+\underline{\frac{\partial}{\partial y}\left(N_2\frac{\partial v^0}{\partial y}\right)}=0$$

$$\delta\psi_x\colon M_{1,x}+M_{6,y}-Q_1+\left(\frac{2}{h}\right)^2R_1-\frac{4}{3h^2}\left(P_{1,x}+P_{6,y}\right)=0$$

$$\delta\psi_y\colon M_{6,x}+M_{2,y}-Q_2+\left(\frac{2}{h}\right)^2R_2-\frac{4}{3h^2}\left(P_{6,x}+P_{2,y}\right)=0$$

$$\delta w^0\colon Q_{1,x}+Q_{2,y}+q-\frac{4}{h^2}\left(R_{1,x}+R_{2,y}\right)-\frac{4}{3h^2}\left(P_{1,xx}+2P_{6,xy}+P_{2,yy}\right)$$

$$+\underline{\frac{\partial}{\partial x}\left(N_1\frac{\partial w^0}{\partial x}+N_6\frac{\partial w^0}{\partial y}\right)+\frac{\partial}{\partial y}\left(N_2\frac{\partial w^0}{\partial y}+N_6\frac{\partial w^0}{\partial x}\right)}=0$$

$$\delta\zeta_z\colon \underline{2M_3-\frac{1}{3}\left(P_{1,xx}+2P_{6,xy}+P_{2,yy}\right)}=0 \tag{17}$$

Specify, at $x = 0$, of a rectangular laminated plate the given force is $\hat{N}_1$, $\hat{N}_6$, $\hat{Q}_1$, $\hat{M}_1$, $\hat{M}_6$, $\hat{R}_1$, $\hat{S}_1$,$\hat{T}_1$ and the boundary conditions are of the form:

$$\delta u^0\colon \hat{N}_1=N_1\left(1+\frac{\partial u^0}{\partial x}\right)$$

$$\delta v^0\colon \hat{N}_6=N_6$$

$$\delta w^0\colon \hat{Q}=N_1\frac{\partial w^0}{\partial x}+N_6\frac{\partial w^0}{\partial y}+\frac{4}{3h^2}\left(P_{1,x}+P_{6,y}\right)+Q_1-\frac{4}{h^2}R_1$$

$$\delta\psi_x\colon \hat{M}_1=M_1-\frac{4}{3h^2}P_1$$

$$\delta\psi_y\colon \hat{M}_6=M_6-\frac{4}{3h^2}P_6$$

$$\delta\zeta_z\colon \hat{R}_3=R_1+\frac{1}{3}\left(P_{1,x}+P_{6,y}\right)$$

$$\delta\left(\frac{\partial w^0}{\partial x}\right)\colon \hat{S}=-\frac{4}{3h^2}\left[P_1\left(\frac{\partial w^0}{\partial x}\right)+P_6\left(\frac{\partial w^0}{\partial y}\right)\right]$$

$$\delta\left(\frac{\partial \zeta_z}{\partial x}\right)\colon \hat{T}=-\frac{1}{3}\left[P_1\left(\frac{\partial \zeta_z}{\partial x}\right)+P_6\left(\frac{\partial \zeta_z}{\partial}\right)\right] \tag{18}$$

It should be noted that Reddy (1984) did not account for the underline terms in eqns (17).

6. Bending analysis

6.1. Result of the linear analysis

In order to validate the present theory, the linear analysis of a symmetric three-layer square laminate (Pagano, 1970; Shu and Sun, 1994) is performed. First these materials of the laminae are assumed to have the following values for the engineering constants:

$$E_3=E_2,\quad E_1=25E_2,\quad G_{12}=G_{13}=0.5E_2,\quad G_{23}=0.2E_2,$$

Table 1
The deflection and stresses of the square cross-ply (0°/90°/0°) plate under sinusoidal loading

a/h		$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$
4	Pagano	1.954	0.720	0.0467	0.291
	Present	1.896	0.667	0.0451	0.211
	Reddy	1.894	0.665	0.0440	0.206
	FSDT	1.710	0.406	0.0308	0.140
10	Pagano	0.743	0.559	0.0275	0.301
	Present	0.716	0.547	0.0267	0.270
	Reddy	0.715	0.546	0.0268	0.264
	FSDT	0.663	0.499	0.0241	0.167
20	Pagano	0.517	0.543	0.0230	0.328
	Present	0.506	0.539	0.0229	0.285
	Reddy	0.506	0.539	0.0228	0.283
	FSDT	0.491	0.527	0.0221	0.175

$\bar{w} = E_2 h^3 w\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)/(q_0 a^3)$, $\bar{\sigma}_x = \sigma_x\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)h^2/(q_0 a^2)$, $\bar{\tau}_{xy} = \tau_{xy}\left(0,0,\frac{h}{2}\right)h^2/(q_0 a^2)$, $\bar{\tau}_{xz} = \bar{\tau}_{xz}\left(0,\frac{b}{2},0\right)h/(q_0 a)$ (Shu and Sun, 1994).

$$v_{23} = v_{31} = v_{12} = 0.25, \quad E_2 = 6.89 \times 10^6 \mathrm{KN/m^3}\left(10^6 \mathrm{psi}\right)$$

Table 1 contains non-dimensionalized deflection and stress for the problems. By comparing these results, it is obvious that the present theory gives the same accuracy results as the Reddy (1984), and the results are in good agreement with the analytical solution of three-dimensional elastic results of Pagano (1970, 1972).

6.2. Cord-rubber laminate nonlinear bending analysis

Here we consider the simply supported rectangular cord-rubber laminated plates. The geometry of the plate with thickness h, length a and width b, and the displacement boundary conditions are shown in Fig. 1. Eqn (19) gives the elastic constants of the cord-rubber lamina.

$$E_1 = 0.617\mathrm{GPa}, \quad E_2 = E_3 = 0.008\mathrm{GPa}, \quad v_{12} = 0.475$$
$$G_{13} = G_{12} = 0.0262\mathrm{GPa}, \quad G_{23} = 0.00233\mathrm{GPa} \tag{19}$$

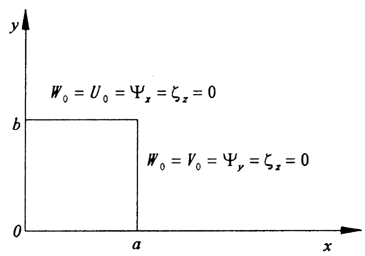

Fig. 1. Displacement boundary conditions of simply supported rectangular laminated plate.

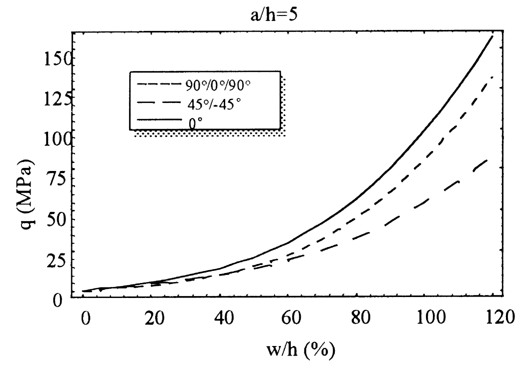

Fig. 2. Deflection and load curves.

Following the Navier solution procedure, we assume the following solution form that satisfies the boundary conditions,

$$U_0 = \sum_{m,n=1}^{\infty} U_{mn}\cos(\alpha x)\,\sin(\beta y) \quad \Psi_x = \sum_{m,n=1}^{\infty} X_{mn}\,\cos(\alpha x)\,\sin(\beta y)$$

$$V_0 = \sum_{m,n=1}^{\infty} V_{mn}\,\sin(\alpha x)\,\cos(\beta y) \quad \Psi_y = \sum_{m,n=1}^{\infty} Y_{mn}\,\sin(\alpha x)\,\cos(\beta y)$$

$$W_0 = \sum_{m,n=1}^{\infty} W_{mn}\,\sin(\alpha x)\,\sin(\beta y) \quad Z_z = \sum_{m,n=1}^{\infty} Z_{mn}\sin(\alpha x)\,\sin(\beta y) \tag{20}$$

Where $\alpha = m\pi/a$, $\beta = n\pi/b$, also we assume that the applied transverse load q can be expanded in the double-Fourier series as

$$q = \sum_{m,n=1}^{\infty} q_{mn}\,\sin(\alpha x)\,\sin(\beta y) \tag{21}$$

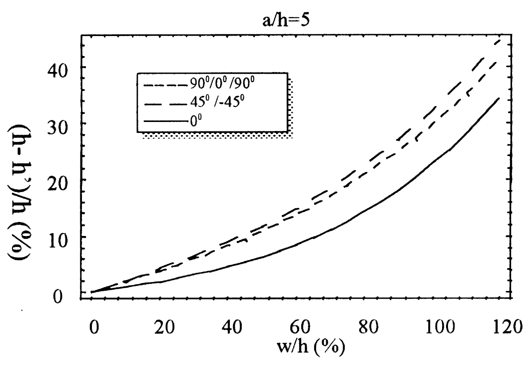

Fig. 3. Thickness variation with deflection.

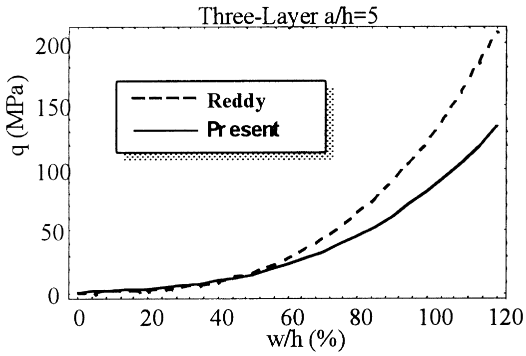

Fig. 4. A contrast to Reddy theory with deflection and loading curves.

Substituting eqns (20) and (21) into displacement fields (5), then substituting them into geometrical nonlinear strain-displacement relation eqn (13), constitutive eqn (14), and using governing eqns (16) and (17), we can obtain the differential equations which include the coefficients U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn}, Z_{mn}. Here are presented numerical results for cord-rubber laminated plate.

As shown in Fig. 2 the deflection of laminated plate and loading relations are obtained by using the present theory. As shown in Fig. 3 the central thickness of the laminated plates change with deflection. The compared results using the present theory and using Reddy simple higher-order shear deformations theory for the case $a/h = 5$ and $a/h = 10$ were given in Figs. 4 and 5. Figs. 6 and 7 show the stress distributions at different planes of laminated plates.

1. Consider relatively thin laminated plates (where $a/h > 10$), the agreement with Reddy's solution is exceptionally good in the region of low deflections (the ratio of the central deflection to the thickness is not greater than 20%).
   With the incease of the ratio value, the difference between the results derived from the presented theory and the Reddy's theory becomes obvious. In particular, when the ratio is greater than 100%, the effect of the normal stress which lead to large deformation of the thickness cannot be neglected. It has been proven that the six-variable geometrical nonlinear laminated theory is more accurate than Reddy's theory to use for dealing with large deformation problems.

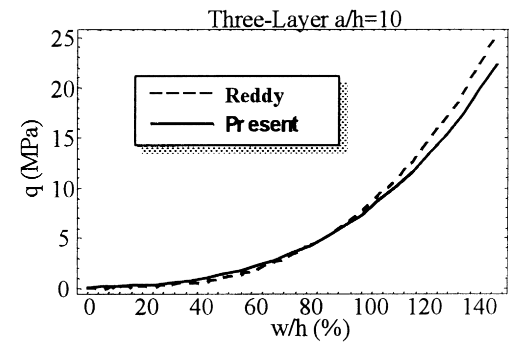

Fig. 5. A contrast to Reddy theory with deflection and loading curves.

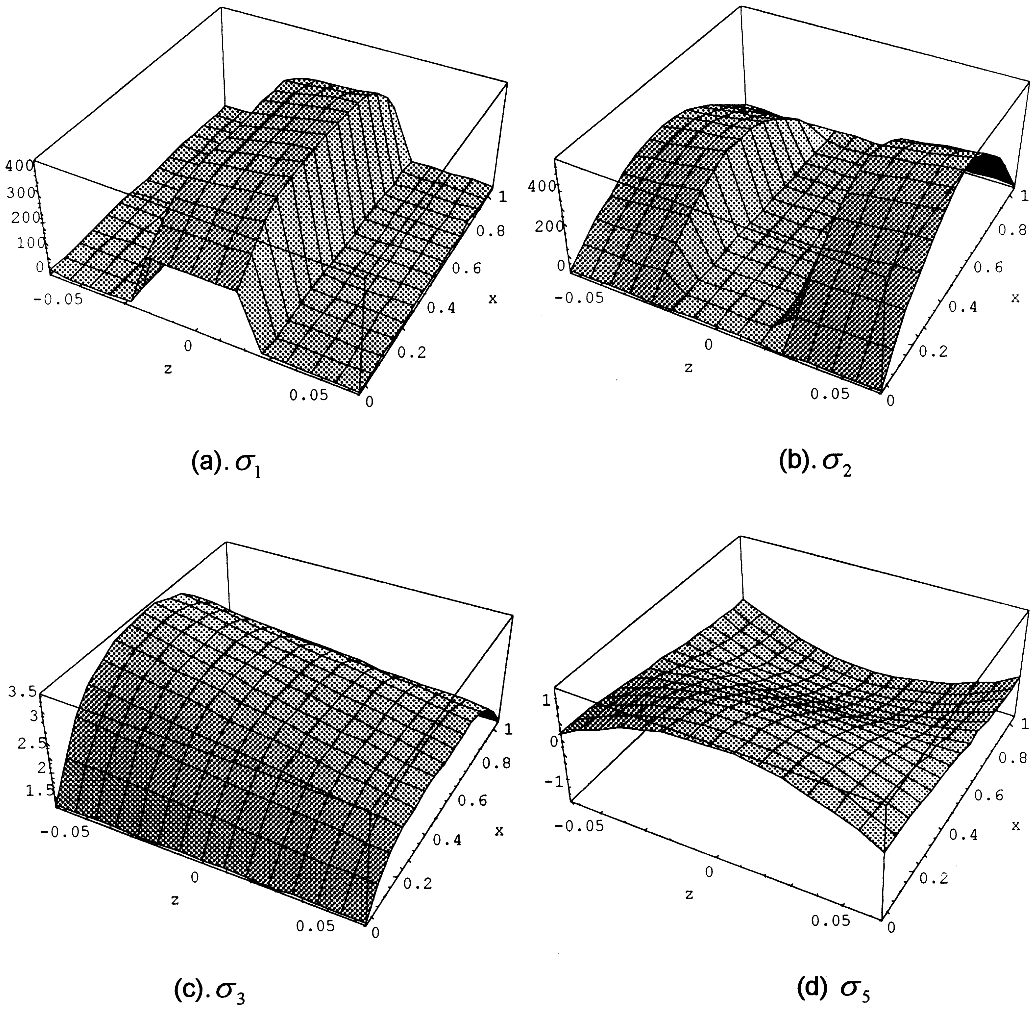

Fig. 6. Stress distributions on the *xz*-plane of 90°/0°/90° laminates (where $y = b/2$, $q_0 = 121$ MPa, $a/h = 5$).

2. Consider relatively thick laminated plates (where $a/h < 5$), the percentage of the deformation of thickness of the center is about 2% even in the range of low deflection ($w/h < 20\%$). Furthermore, when the ratio of deflection to thickness increases to 80% the deformation of thickness increases to more than 10%. The solutions in Figs. 3–5 reveal that the present six-variable geometrical nonlinear laminated theory, which includes three-dimensional information and modified Von Karman geometrical nonlinear relations, is more fit for thick laminated plates analysis and large deformation analysis.
3. The transverse shear strains are parabolic distributions through thickness of the plate and the transverse stresses are continuous across each layer interface.
4. Results shown in Fig. 3 indicate that the thickness of the plate varies nonlinearly with the deflection.

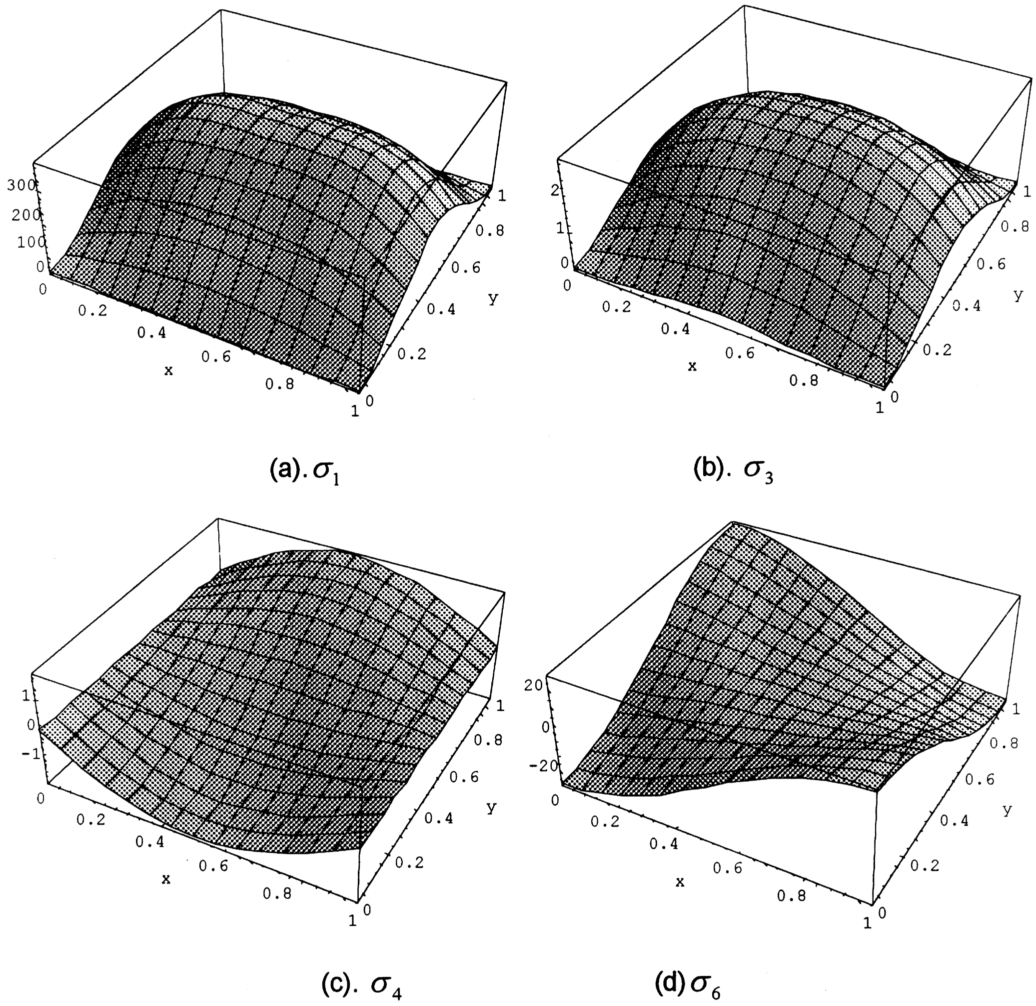

Fig. 7. Stress distributions on the middle plane of 90°/0°/90° laminates (where $q_0 = 121$ MPa, $a/h = 5$).

7. Conclusion

A six-variable geometrical nonlinear shear deformation laminated theory is presented in which normal stress and strain distribution can be calculated. By considering some affective factors that were neglected under the finite deformation condition, an improved Von Karman deformation-strain relation is used for large deformation analysis.

By comparing the results obtained with Reddy's simple higher-order shear deformation theory, it is obvious that the six-variable higher-order geometrical nonlinear shear deformation laminated theory gives a closer approximation to the behavior of laminated plates. This is especially true in the case of relative thick laminates where the effects of the normal components of stress and strain could not be neglected, and large deformation analysis.

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